

# Control

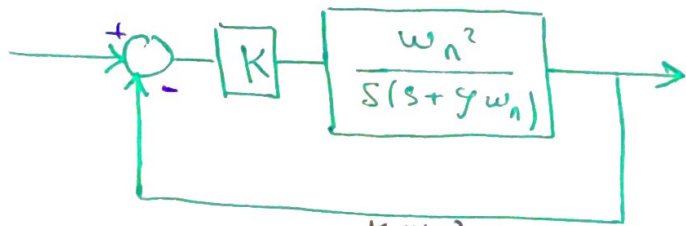
15/10/2015

الخميس

م. خطاب

الصفحة 3

## Root Locus



$$\begin{aligned} \text{C.L.T.F} &= \frac{\cancel{K\omega_n^2} / s(s + \gamma\omega_n)}{1 + \cancel{K\omega_n^2} / s(s + \gamma\omega_n)} \\ &= \frac{K\omega_n^2}{s^2 + \gamma\omega_n s + K\omega_n^2} \end{aligned}$$

## Example

$$\text{O.L.T.F} = \frac{K}{s(s+2)}$$

$$\text{C.L.T.F} = \frac{K}{s^2 + 2s + K}$$

$$K=1 \Rightarrow (s+1)^2$$

$$\begin{aligned} K=0.5 \Rightarrow s_{1,2} &= \frac{-2 \pm \sqrt{4-2}}{2} \\ &= -1 \pm \frac{1}{\sqrt{2}} \end{aligned}$$

## Example:

$$\text{O.L.T.F} = \frac{K Z(s)}{P(s)}$$

$$\begin{aligned} \text{C.L.T.F} &= \frac{K Z(s) / P(s)}{1 + K Z(s) / P(s)} \\ &= \frac{K Z(s)}{P(s) + K Z(s)} \end{aligned}$$

at  $K=0$

C.L. poles = O.L. poles

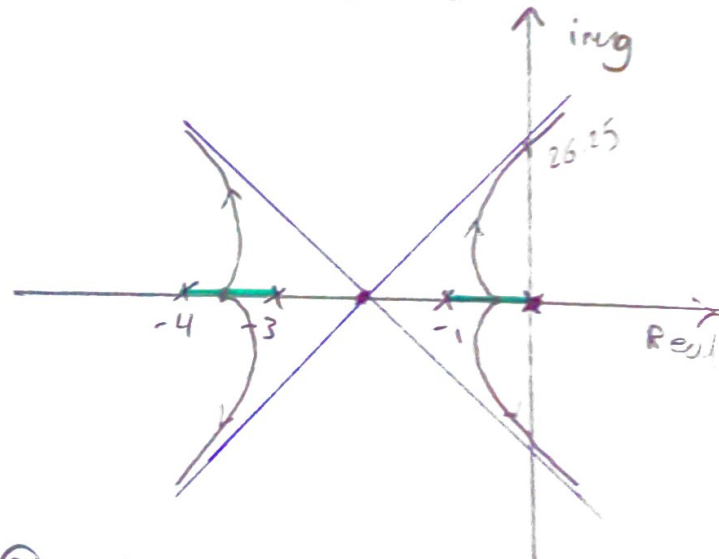
at  $K=\infty$

O.L. poles = O.L. zeros

$$Q: G H(s) = \frac{K}{s(s+1)(s+3)(s+4)}$$

① on the sketch, determine (plot) open loop poles and zeros

$n_p = \checkmark, n_z = \checkmark$



② Determine the parts of the real axis that belong to root locus

The parts of the real axis located at the left of odd number of o.l. poles and zeros belong to the root locus

$\Rightarrow$  Turn over

### ③ Asymptotic lines

$$\# = n_p - n_z$$

$$\sigma = \frac{\sum P - \sum Z}{n_p - n_z}$$

$$\theta = \frac{(2l+1) \times 180}{n_p - n_z} \quad l = 0, 1, 2$$

$$\# = 4$$

$$\sigma = \frac{-1-3-4}{4} = -2$$

$$\theta = 45, 135, 225, 315$$

$$= \pm 45, \pm 135$$

### ④ Breaking points

$$K = \frac{-1}{G_H(s)}$$

$$\frac{dK}{ds} = 0$$

⇒ solve for  $s$

for higher order systems

$$K = \frac{-1}{G_H(s)}$$

S	-6	-5	-4	-3	-2	-1	0
K							

نقاط التماس  $K$

### ⑤ Range of $K$ for stability

\* use Routh Array

$$1 + G_H(s) = 0$$

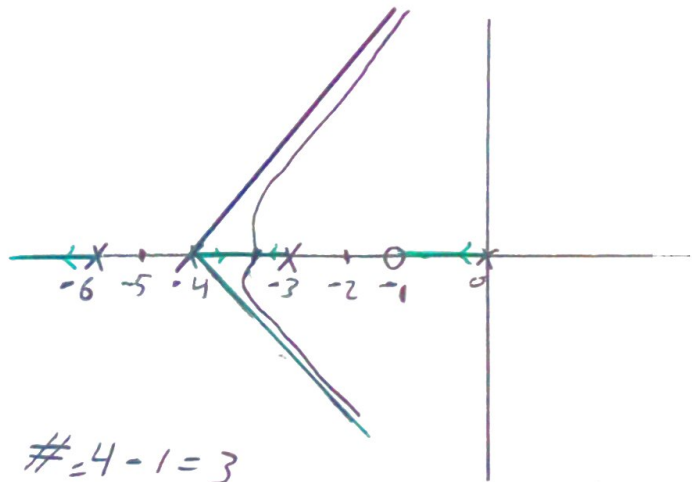
$$s(s+1)(s+3)(s+4) + K = 0$$

$$s^4 + 8s^3 + 19s^2 + 12s + K = 0$$

$s^4$	1	19	$K$
$s^3$	8	12	
$s^2$	17.5	$K$	
$s^1$	$\frac{12-8K}{17.5}$		
$s^0$	$K > 0$		

$$K = 26.25 \leftarrow \text{critical value}$$

$$Q: G_H(s) = \frac{K(s+1)}{s(s+6)(s+3)(s+4)}$$



$$\# = 4 - 1 = 3$$

$$\sigma = \frac{-6-3-4+1}{3} = -4$$

$$\theta = \frac{(2l+1) \times 180}{3} = 60, 180, -60$$

### ④ Breaking Points

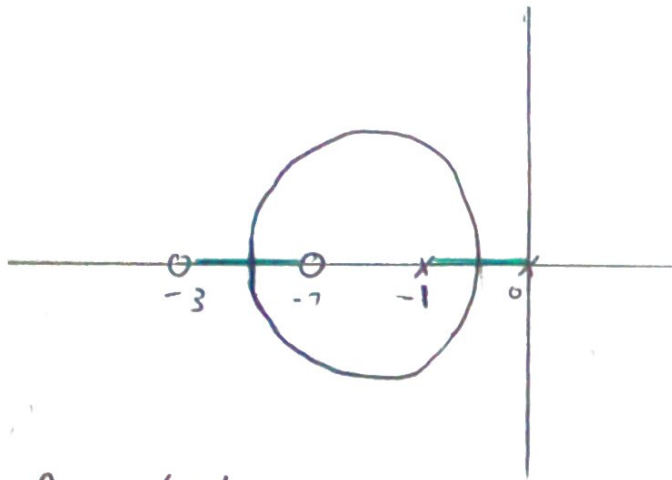
S	-3.5
K	

$$K = - \frac{s(s+3)(s+4)(s+6)}{s+1}$$

### ⑤ Range of $K$

Routh (do it yourself)

$$Q: GH(s) = \frac{K(s+2)(s+3)}{s(s+1)}$$



Asymptotes = 0

- Breaking points (Maximum)

out: -1

s	--	-0.5	--
K			

$$K = \frac{-1}{GH(s)} = \frac{-s(s+1)}{(s+2)(s+3)}$$

in: -2 : -3 Minimum

s		-2.5	
K			

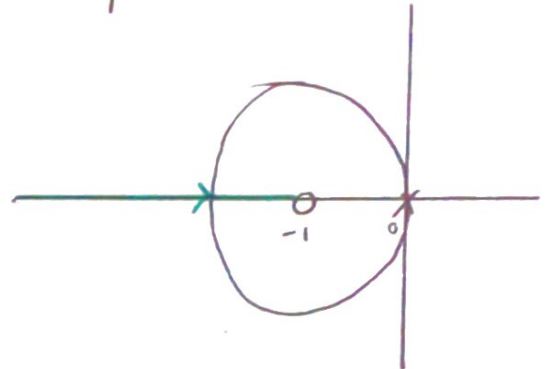
Report (optional)

Prove that the root locus must be circle

hint: Angle Condition

$$Q: GH(s) = \frac{K(s+1)}{s^2}$$

report



```
num=[1 5 6];
den=[1 1 0];
G=tf(num,den);
rlocus(G)
```



MatLab